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HYDRODYNAMIC CALCULATIONS OF LAYERED SEAMS ON THE BASIS OF MODIFIED RELATIVE PERMEABILITIES

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The possibility of reducing the dimensionality of the problem of two-phase filtration in layered seams by introducing modified phase permeabilities instead of initial relative permeabilities, which are coefficients of the initial system of equations within the framework of the Barkley–Leverett model, is studied. Modified permeabilities are proposed for the case where the relative permeabilities of each phase are represented by analytical dependences for individual interlayers. One-dimensional numerical calculations with these permeabilities are in good agreement with the solution of the two-dimensional problem.

We consider a two-dimensional (x, z) problem of two-phase displacement of oil by water in a layered nonuniform seam between two galleries with a given pressure difference. We assume that the fluids are incompressible, capillary and gravitational forces are absent, and the flow is described by the Barkley–Leverett model. The mathematical formulation of this problem [1] with isothermal filtration has the following form:

$$\operatorname{div}(K_\Sigma \operatorname{grad} P) = 0, \qquad \operatorname{div}(F K_\Sigma \operatorname{grad} P) = m\,\frac{\partial S}{\partial t},$$

$$K_\Sigma = K(z)(K_{\rm w}(S)/\mu_{\rm w} + K_{\rm oil}(S)/\mu_{\rm oil}), \qquad F = K(z)K_{\rm w}(S)/(\mu_{\rm w} K_\Sigma).$$

The initial and boundary conditions for the pressure P and water saturation S are $P|_{x=0} = P_1$, $P|_{x=L} = P_2$, $S|_{x=0} = S_{\max} = S^*$, and $S|_{t=0} = S_{\min} = S_*$; the adjoint conditions for the pressure and vertical fluxes of the phases at the boundaries of interlayers composing the layered seam are $P^+ = P^-$, $v^+_{{\rm w},z} = v^-_{{\rm w},z}$ and $v^+_{{\rm oil},z} = v^-_{{\rm oil},z}$; the conditions of impermeability of the subface and roof of the seam are $\partial P/\partial z|_{z=0,{\rm oil}} = 0$. Here $K(z)$ is the absolute permeability of the seam consisting of hydrodynamically related, horizontally uniform interlayers with different absolute permeabilities [the dependence $K(z)$ obeys the probability distribution law with a density $f(k)$]; $K_{\rm w}(S)$ and $K_{\rm oil}(S)$ are the relative permeabilities of water and oil, respectively, which are determined by core samples; $\mu_{\rm w}$ and $\mu_{\rm oil}$ are the viscosities of the corresponding phases, H is the power (thickness) of the seam; L is the distance between the outlet and exploitation galleries, m is the porosity, and S^* is the water saturation in the outlet gallery.

The problem was solved numerically using a finite-difference scheme (alternatively triangular method [2]). The calculations were performed for a seam consisting of five uniform interlayers of identical thickness. The function $K(z)$ was specified as follows: uniform distribution over the seam thickness, exponential distribution, and Maxwellian distribution.

We consider the oil recovery factor η as a function of the time of seam exploitation t or pumped porous volumes τ (Fig. 1). Since the interlayers in the initial seam may be located in different order along the vertical, the dependence $\eta(\tau)$ yields a set of curves significantly different from each other. Figure 1 shows the limiting values of this set. Curve 2 refers to a seam in which the neighboring interlayers possess the greatest and the least values of absolute permeability. Powerful vertical overflow arises between the interlayers, which ensures a greater degree of oil displacement by water. Curve 3 corresponds to a seam with isolated interlayers (interlayer boundaries are impermeable). In this case, there is no overflow, and the least displacement of oil by water is observed. The

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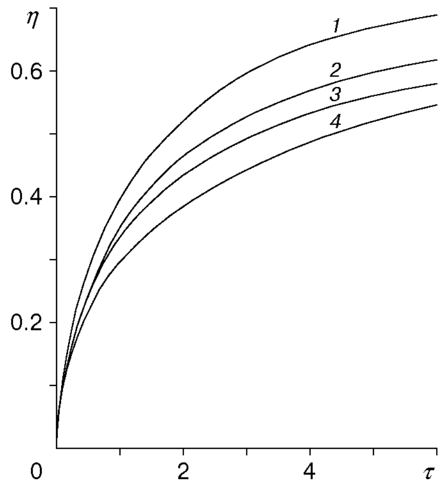

Fig. 1. Oil recovery factor η as a function of τ [uniform distribution $K(z)$ over the interlayers; $K_{\rm w}(S)$ and $K_{\rm oil}(S)$ are linear functions]: curve 1 refers to the solution C, curves 2 and 3 show the upper and lower boundaries of the reference solutions A_i, respectively, curve 4 refers to the solution B.

calculations were performed for the following values of the absolute permeability of interlayers: $K_1 = 0.1$ darcy, $K_2 = 0.3$ darcy, $K_3 = 0.5$ darcy, $K_4 = 0.7$ darcy, and $K_5 = 0.9$ darcy. Here $K(z)$ is a piecewise-continuous function; the coefficient of variation of layered inhomogeneity over the seam thickness $V = 0.55$ is close to the maximum value for the prescribed uniform distribution.

The relative permeabilities were written in the form

$$K_{\rm w}(S) = K_{{\rm w}_0}(S_{\rm m}(S)), \qquad K_{\rm oil}(S) = K_{{\rm oil}_0}(1 - S_{\rm m}(S)), \qquad S_{\rm m}(S) = (S - S_*)/(S^* - S_*), \tag{1}$$

where $S_{\rm m}$ is the "movable" water. The numerical solution of the two-dimensional (x, z) problem obtained with the use of phase permeabilities (1) and the above-mentioned absolute permeabilities K_j $(j = \overline{1, 5})$ of the interlayers is assumed to be the reference solution and is designated as A_i. Curves 2 and 3 in Fig. 1 limit the set of the reference solutions A_i from above and from below.

It follows from the numerical results that similar positions of these curves are observed for other distribution laws of $K(z)$, i.e., for each distribution law, there is a corresponding set of reference solutions A_i.

We reduce the dimensionality of the initial two-dimensional two-phase problem. Instead of it, we solve a one-dimensional problem within the framework of the Barkley–Leverett model. The absolute permeability is assumed to be equal to its mean value over the seam thickness $K^* = \dfrac{1}{H}\displaystyle\int_0^H K(z)\,dz$, and the relative permeabilities are determined using the same dependences (1) as in the two-dimensional problem. The resultant numerical solution is called the solution C. The calculation results are plotted in Fig. 1 (curve 1).

The relative permeabilities in the one-dimensional problem are further used in the form

$$K'_{\rm w}(S) = K_{\rm w}(S)A(S), \qquad K'_{\rm oil}(S) = K_{\rm oil}(S)B(S), \tag{2}$$

where $A(S)$ and $B(S)$ are correction coefficients, which will be found using the method described below. The numerical solution of this one-dimensional problem will be called the solution B (curve 4 in Fig. 1).

We consider a two-dimensional two-phase flow in a layered seam. The initial relative permeabilities are defined in the form of (1). Since the motion of the maximum water saturation S^* from the outlet gallery to the exploitation gallery occurs in the numerical solution of the two-phase filtration problem only in the case of linear dependences $K_{\rm w}(S)$ and $K_{\rm oil}(S)$ [6], we may speak of complete displacement of movable oil in the most permeable interlayers, where the function $S(x, z)$ reached the value S^*, i.e., of strong "cusping" [3]. Based on this fact, we assume that the flow has a jetlike character [4, 5] and consider a jet flow in a layered seam instead of two-dimensional two-phase filtration. It is assumed that the seam consists of many isolated interlayers whose properties are constant along the horizontal direction and variable along the vertical one. Water displaces oil in interlayers and moves in jets of various lengths. The motion is faster in interlayers with a greater permeability. Therefore, the interlayers in each vertical section may be united into two zones: the zone of water of thickness H_{w}, where $S(x,z)=S^*$, and the zone of oil of thickness H_{oil}, where $S(x,z)=S_*$. It follows from these assumptions that, in each vertical section of the seam, one can find an absolute permeability $\bar{K}$ such that the following relation is valid: $H=H_{\mathrm{w}}+H_{\mathrm{oil}}$; here $H_{\mathrm{w}}=\sum_{i=1}^{n_1} H_i$ for $K_i>\bar{K}$ and $H_{\mathrm{oil}}=\sum_{j=1}^{n_2} H_j$ for $K_j\leqslant\bar{K}$ (K_i, K_j, H_i, and H_j are the absolute permeabilities and thicknesses of the interlayers; n_1 and n_2 are the numbers of interlayers in the seam).

We consider a part of volume of seam pores ΔV_{w} of thickness H_{w} and length Δx, which are filled by water at the time t $[S(x,z)=S^*]$. We also consider the total volume of pores ΔV of thickness H and length Δx. Obviously, we have $\Delta V=H\Delta xm$ and $\Delta V_{\mathrm{w}}=H_{\mathrm{w}}\Delta xmS^*+(H-H_{\mathrm{w}})\Delta xmS_*$. We find the water saturation at each point x: $\tilde{S}(x)=\lim_{\Delta x\to 0}(\Delta V_{\mathrm{w}}/\Delta V)$ [6]. Obviously, this value is equal to the mean water saturation over the seam thickness in a jet flow: $\tilde{S}(x)=\dfrac{1}{H}\displaystyle\int_0^H S(x,z)\,dz$.

Taking into account the above reasoning, we can consider a jet flow in a layered seam as a one-dimensional two-phase flow with a mean value K^* and water saturation $S(x)$ close to $\tilde{S}(x)$. To write the continuity equations for one-dimensional two-phase filtration, in this case, it is necessary to assume that there is a rather large number of different interlayers in the layered seam so that H_{w} and $\tilde{S}(x)$ are continuous functions, which is what we do since we consider a continuous distribution $K(z)$. In these one-dimensional continuity equations, we use averaged values of water and oil permeabilities under the assumption of the jetlike character of the flow in interlayers [4, 5] and validity of the relation $H=H_{\mathrm{w}}+H_{\mathrm{oil}}$. With regard for this fact, the continuity equations include the absolute permeability K^* and the modified phase permeabilities (2), which have the form

$$K'_{\mathrm{w}}(S)=K_{\mathrm{w}_0}(S_{\mathrm{m}}(S))\bar{K}_{\mathrm{w}}(S)/K^*,\qquad K'_{\mathrm{oil}}(S)=K_{\mathrm{oil}_0}(1-S_{\mathrm{m}}(S))\bar{K}_{\mathrm{oil}}(S)/K^*. \tag{3}$$

The mean permeabilities $\bar{K}_{\mathrm{w}}(S)$ and $\bar{K}_{\mathrm{oil}}(S)$ are obtained using the function $f(k)$, which is the probability density of the distribution $K(z)$ $[a\leqslant K(z)\leqslant b]$:

$$\bar{K}_{\mathrm{w}}(S)=\int_{\bar{K}}^{b} kf(k)\,dk\Big/\int_{\bar{K}}^{b} f(k)\,dk,\qquad \bar{K}_{\mathrm{oil}}(S)=\int_{a}^{\bar{K}} kf(k)\,dk\Big/\int_{a}^{\bar{K}} f(k)\,dk. \tag{4}$$

The value of $\bar{K}$ is found from the given water saturation $S(x)$ by the solving numerically the equation

$$1-S_{\mathrm{m}}(S)=\int_{a}^{\bar{K}} f(k)\,dk. \tag{5}$$

For the initial linear permeabilities $K_{\mathrm{w}}(S)$ and $K_{\mathrm{oil}}(S)$ [see (1)], the coefficients $A(S)$ and $B(S)$ are written in the form

$$A(S)=\bar{K}_{\mathrm{w}}(S)/K^*,\qquad B(S)=\bar{K}_{\mathrm{oil}}(S)/K^*. \tag{6}$$

For an arbitrary distribution of the function $K(z)$, the modified permeabilities $K'_{\mathrm{w}}(S)$ and $K'_{\mathrm{oil}}(S)$ are found numerically by formulas (3)–(6). In the case of a uniform distribution of the function $K(z)$ over the thickness, we can easily obtain the analytical dependences

$$K'_{\mathrm{w}}(S)=K_{\mathrm{w}_0}S_{\mathrm{m}}(S)[1+V\sqrt{3}(1-S_{\mathrm{m}}(S))],\quad K'_{\mathrm{oil}}(S)=K_{\mathrm{oil}_0}(1-S_{\mathrm{m}}(S))[1+V\sqrt{3}(1-S_{\mathrm{m}}(S))], \tag{7}$$

where V is the coefficient of variation of layered nonuniformity.

The calculations of the one-dimensional problem of two-phase filtration by formulas (7) are plotted in Fig. 1. Curves 4 and 1 are the lower and upper boundaries of these solutions, respectively. Thus, the reference solutions A_i are within the range of the one-dimensional solutions B and C.

It follows from Eqs. (3)–(7) that the modified permeabilities are obtained from the linear relative permeabilities (1) by multiplication by the coefficients $A(S)$ and $B(S)$. In considering a layered seam in which the values of $K(z)$ remain almost unchanged (i.e., the seam is extremely uniform), we have $A(S)=B(S)=1$. The latter equality is obvious, since the permeability of the water zone in such a seam is equal to the permeability of the oil zone and to the mean permeability of the seam itself. In this case, we have $K'_{\mathrm{w}}(S)=K_{\mathrm{w}}(S)$ and $K'_{\mathrm{oil}}(S)=K_{\mathrm{oil}}(S)$, which corresponds to the limiting case (uniform seam).

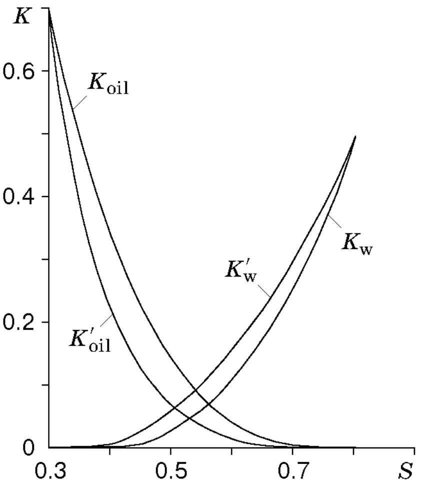

Fig. 2

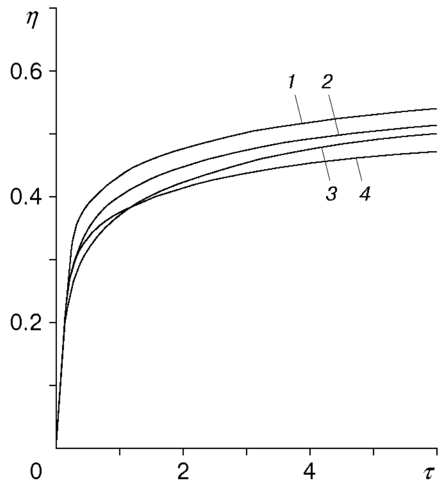

Fig. 3

Fig. 2. Dependences $K_{\rm oil}(S)$, $K'_{\rm oil}(S)$, $K_{\rm w}(S)$, and $K'_{\rm w}(S)$ [uniform distribution $K(z)$ over the interlayers; $K_{\rm w}(S)$ and $K_{\rm oil}(S)$ are cubic functions].

Fig. 3. Dependence $\eta(\tau)$ [uniform distribution $K(z)$ over the interlayers; $K_{\rm w}(S)$ and $K_{\rm oil}(S)$ are cubic functions] (notation the same as in Fig. 1).

Nevertheless, laboratory studies show that $K_{\rm w}(S)$ and $K_{\rm oil}(S)$ are most often nonlinear functions. Sometimes they are assumed to be quadratic or cubic parabolas

$$K_{\rm w}(S) = K_{{\rm w}_0}(S_{\rm m}(S))^{\alpha}, \qquad K_{\rm oil}(S) = K_{{\rm oil}_0}(1 - S_{\rm m}(S))^{\beta} \qquad (\alpha = 2, 3; \quad \beta = 2, 3). \tag{8}$$

In this case, the assumption about the jetlike character of the flow in the layered seam is impossible, since, in the case of nonlinear dependences $K_{\rm w}(S)$ and $K_{\rm oil}(S)$, there is no motion of the section of the maximum water saturation S^* [6] and complete displacement in each interlayer of the seam. Therefore, the modified permeabilities (3) cannot be obtained by averaging.

We solve a one-dimensional problem with modified permeabilities of the form (2), which are obtained by correction of the initial nonlinear dependences $K_{\rm w}(S)$ and $K_{\rm oil}(S)$ (8) with the help of the coefficients $A(S)$ and $B(S)$. In the case $A(S) = B(S) = 1$, we have a one-dimensional solution C (see Fig. 1). If these coefficients are taken in the same form as for the linear case [see (4)–(6)], we obtain a one-dimensional solution B (see Fig. 1). Figure 2 shows the dependences $K_{\rm w}(S)$, $K_{\rm oil}(S)$, $K'_{\rm w}(S)$, and $K'_{\rm oil}(S)$ (3) in the case $\alpha = \beta = 3$ for a uniform distribution $K(z)$ over the interlayers. The location of the curves in Fig. 2 is the same as in the case of linear dependences $K_{\rm w}(S)$ and $K_{\rm oil}(S)$ [4].

Figure 3 shows the dependences $\eta(\tau)$ for $\alpha = \beta = 3$. As in the linear case (see Fig. 1), the reference solutions A_i lie within the range of one-dimensional solutions B and C. In addition, in the above-considered case of nonlinear dependences $K_{\rm w}(S)$ and $K_{\rm oil}(S)$, the modified permeabilities (2) coincide with the initial permeabilities in passing to the limiting uniform seam, since $A(S) = B(S) = 1$. For the problem considered, the correction coefficients were first proposed in [7]. Thus, the correction coefficients for the jet flow are also applicable in the nonlinear case.

We consider a more general case of setting the dependences $K_{\rm w}(S)$ and $K_{\rm oil}(S)$ in a layered seam. It is known that the relative permeabilities $K_{\rm w}(S)$ and $K_{\rm oil}(S)$ for different interlayers are often described by different analytical dependences. It is rather difficult to construct the common dependences $K'_{\rm w}(S)$ and $K'_{\rm oil}(S)$ for the whole seam. We consider the case of permeabilities set in the form $K_{\rm w}(S) = K_{{\rm w}_0}(S_{\rm m}(S))^{\alpha_i}$ and $K_{\rm oil}(S) = K_{{\rm oil}_0}(1 - S_{\rm m}(S))^{\beta_i}$, where α_i and β_i are constants depending on the number i of the interlayer ($\alpha_i, \beta_i \geqslant 1$). By analogy with the previous one-dimensional solutions C, we use the values of permeabilities averaged over the seam thickness:

$$\langle K_{\rm w}(S)\rangle = \frac{1}{H}\sum_{i=1}^{n} H_i K_{\rm w}(S), \qquad \langle K_{\rm oil}(S)\rangle = \frac{1}{H}\sum_{i=1}^{n} H_i K_{\rm oil}(S). \tag{9}$$

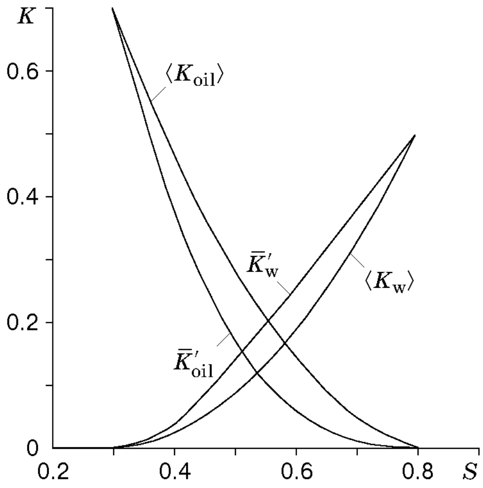

Fig. 4

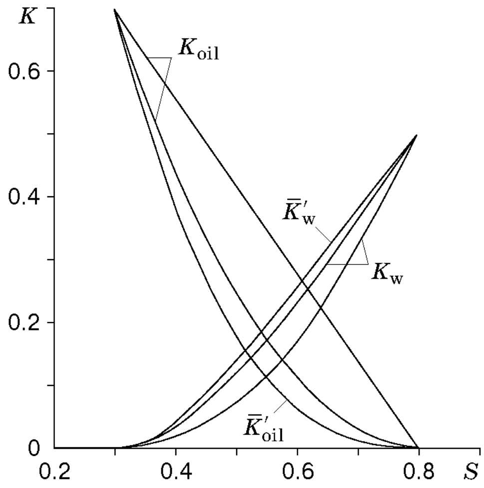

Fig. 5

Fig. 4. Mean permeabilities $\langle K_{\rm w}\rangle$ and $\langle K_{\rm oil}\rangle$ calculated by (9) and modified permeabilities $\bar{K}'_{\rm w}$ and $\bar{K}'_{\rm oil}$ calculated by (10).

Fig. 5. Permeabilities $K_{\rm w}$ and $K_{\rm oil}$ obtained using core samples and modified permeabilities $\bar{K}'_{\rm w}$ and $\bar{K}'_{\rm oil}$ calculated by (10).

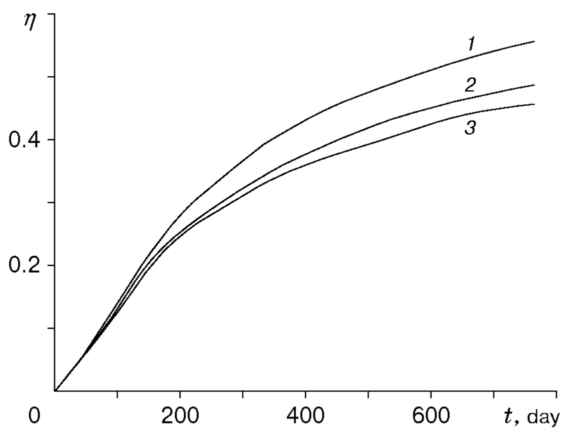

Fig. 6. Distribution of the oil recovery factor versus the time of seam exploitation [uniform distribution $K(z)$ over the interlayers; $K_{\rm w}(S)$ and $K_{\rm oil}(S)$ are nonlinear functions; $V = 0.55$]: curve 1 refers to the solution C, curve 2 shows the reference solution A, curve 3 refers to the solution B.

In addition, another, more complicated approach is used:

$$\bar{K}'_{\rm w}(S) = \langle K_{\rm w}(S)\rangle A(S), \qquad \bar{K}'_{\rm oil}(S) = \langle K_{\rm oil}(S)\rangle B(S). \tag{10}$$

The dependences $\bar{K}'_{\rm w}(S)$ and $\bar{K}'_{\rm oil}(S)$ are obtained by correction of permeabilities averaged over the thickness with the help of the coefficients $A(S)$ and $B(S)$ (4)–(6). These coefficients are obtained for the initial layered seam under the assumption of the jetlike character of displacement for a particular case of linear dependences $K_{\rm w}(S)$ and $K_{\rm oil}(S)$, which are identical for the entire layered seam.

As an example, we again consider a seam consisting of five interlayers of identical thickness but with different permeabilities: $K_{\rm w}(S) = K_{{\rm w}_0}(S_{\rm m}(S))^2$ for $i = 1$, 2, 4, and 5, $K_{\rm w}(S) = K_{{\rm w}_0}(S_{\rm m}(S))^{1.5}$ for $i = 3$, $K_{\rm oil}(S) = K_{{\rm oil}_0}(1 - S_{\rm m}(S))^2$ for $i = 1, 2, 4$, and 5, and $K_{\rm oil}(S) = K_{{\rm oil}_0}(1 - S_{\rm m}(S))$ for $i = 3$.

The mean permeabilities (9) constructed for this seam are shown in Fig. 4, and the modified mean (10) and experimental permeabilities are plotted in Fig. 5. Figure 6 shows the dependence of the oil recovery factor versus the time of seam exploitation in the case of one-dimensional displacement [by Eqs. (9) and (10)] and also in the case of a two-dimensional profile flow (reference solutions A_i). The results obtained are similar to those shown in Figs. 1 and 3. The reference solutions A_i are located between two one-dimensional solutions B and C.

Numerical calculations with different combinations of the functions $K_{\rm w}(S)$ and $K_{\rm oil}(S)$ for interlayers of the initial layered seam with a uniform distribution over the thickness and also with different distributions showed that expressions for the modified permeabilities (2) based on results obtained for a jet flow may be used in calculations. This allows one to used the calculation results of the simplest (jetlike) displacement to analyze more complicated cases of displacement of oil by water in layered seams.

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